

Titles and abstracts

Contemporary glimpses on Grothendieck's mathematics post-IHES

Toulouse, 4-6 December 2024

Bruno Klingler

- **Title:** *Recent progress on Hodge loci*
- **Abstract:** Given a quasi projective family S of complex algebraic varieties, its Hodge locus is the locus of points of S where the corresponding variety admits exceptional Hodge classes (conjecturally: exceptional algebraic cycles). In this talk I will survey the many recent advances in our understanding of such loci, as well as the remaining open questions.

Sébastien Maronne

- **Title:** *Variations in Grothendieck's mathematical style (joint work with Bertrand Toen)*
- **Abstract:** In this talk based on the exhibition, we compare Grothendieck's discourse on his mathematical style in *Récoltes et Semailles* with his mathematical writings, considering texts from 1955-1970 but especially those from the period 1973-1986 during which he was professor at the University of Montpellier. Our aim is to distinguish, following a classical path, history from memory, in order to describe as objectively as possible Grothendieck's mathematical practice. Finally, we will allude to the birth and the reception of the theory of motives.

Florian Naef

- **Title:** *Kashiwara-Vergne, volume forms and simple homotopy types*
- **Abstract:** I will give a very biased overview of some aspects of the Kashiwara-Vergne problem. The original Kashiwara-Vergne problem is a question about finite dimensional Lie algebras, involving certain volume forms. The problem can then be translated into a question about braids on the plane. This in turn admits a generalization in arbitrary dimensions

and is related to string topology. This purely topological avatar of a variant of the Kashiwara-Vergne problem can be seen as studying the simple homotopy type of a space, which is a kind of "non-commutative" volume form. To bring the story full circle, out of such a "non-commutative" volume form on a space, one constructs volume forms on (derived) representation varieties. This is based on joint works with Pavel Safronov.

Joost Nuiten

- **Title:** *Formal integration of partition Lie algebroids*
- **Abstract:** Lie algebroids typically appear in differential geometry and algebraic geometry over \mathbb{Q} as mutual generalisations of foliations and Lie algebras, and as the infinitesimal analogues of Lie groupoids. I will discuss a homotopy-theoretic refinement of these objects, known as partition Lie algebroids, adapted to schemes over a more general base ring. A typical example is the relative tangent bundle of a map of schemes, encoding the infinitesimal foliation by the fibres of the map. Conversely, every partition Lie algebroid on a scheme X has a formal leaf space, given by a formal (derived) stack. Based on joint work with Lukas Brantner and Kirill Magidson.

Mauro Porta

- **Title:** *Homotopy theory of Stokes data and derived moduli*
- **Abstract:** Stokes data are the combinatorial counterpart of irregular meromorphic connections in the Riemann-Hilbert correspondence. In dimension 1, they have been studied by Deligne and Malgrange. In higher dimensions the situation is intrinsically more complicated, but the solution of Sabbah's resolution conjecture by Mochizuki and Kedlaya unlocked a much deeper understanding of the Stokes phenomenon. In recent work with Jean-Baptiste Teyssier, we develop a framework to define and study Stokes data with coefficients in any presentable stable infinity category. As an application, we construct a derived moduli stack parametrizing Stokes data in arbitrary dimensions, extending all previously known representability results. One fundamental input is a finiteness theorem for the stratified homotopy types of algebraic and compact real analytic varieties that we obtained in collaboration with Peter Haine and that extends finiteness theorems for the underlying homotopy types of Lefschetz-Whitehead, Lojasiewicz and Hironaka.

Marcy Robertson

- **Title:** *Configurations of the Punctured Plane, Fibre Sequences, and genus zero solutions to the Kashiwara-Vergne Problem*
- **Abstract:** The classical Kashiwara-Vergne problem seeks explicit constructions of basis-conjugating automorphisms for free Lie algebras that satisfy specific properties, such as ensuring that $F(e^x e^y) = e^{x+y}$. These constructions are often complex and traditionally rely on techniques from quantized deformation theory. Recently, work of Alekseev, Kawazumi, Kuno, and Naef introduced the notion of a higher genus Kashiwara-Vergne problem which illuminates a deep connection between the classical Kashiwara-Vergne problem and the Goldman-Turaev Lie bialgebras of string topology. In this talk, we aim to present a "tower" of configuration spaces and show that specific splittings in this tower correspond to those genus zero Kashiwara-Vergne solutions that can be constructed from Drinfeld associators. In addition, we will explore how this tower of configuration spaces aligns with the genus zero model of the ideal Teichmüller towers that Grothendieck proposed in his *Esquisse d'un Programme*. We will conclude by giving an explicit description of an action of the Grothendieck-Teichmüller group on our tower of genus zero Kashiwara-Vergne solutions and describe some conjectures for extending this work to higher genus. This talk will include parts of joint work with Zsuzsanna Dancso, Iva Halacheva, Guillaume Laplante-Anfossi, and Chandan Singh.

Damian Roessler

- **Title:** *A survey of the correspondence between Grothendieck and Serre*
- **Abstract:** We will give an overview of the mathematical content of the Grothendieck-Serre correspondence, explaining how it relates to the famous SGA seminars held at the IHES in the 1960s.

Leila Schneps

- **Title:** *Grothendieck's Esquisse d'un Programme and the Grothendieck-Teichmüller tower*
- **Abstract:** In the first part of this talk, we will explain the ideas expressed by Grothendieck in parts 2 and 3 of his famous *Sketch of a Programme*, concerning the tower of moduli spaces of curves with marked points and the Galois action on their profinite fundamental groups. Then we will explain what parts of this program have been realized, and what still remains open. We will end with some astonishing connections between the linearized version of the programme (pro-unipotent completions and Lie algebras) and some aspects of number theory, in particular multiple zeta values and modular forms.

Jean-Jacques Szczeciniarz

- **Title:** *Philosophical reflections on the concept of duality*
- **Abstract:** I propose to open the question of whether we can give a unitary concept of duality.

I will proceed in two ways. Starting with an analysis of Serre's duality theorem, supplemented by the analyses of Hartshorne (Residues and duality), I will ask whether it is possible to derive a conceptual form that can be absorbed into philosophical reflection. Secondly, to explore the first question further, I will trace the history of duality theorems back to the recent book "Duality in 19th and 20th century Mathematical thinking"), asking whether the latest form analyzed sheds light on and deepens the first formulations, which appear in two chosen steps, particularly significant: Poincaré's theorem, categorical theorems, "On certain aspects of homological algebra" (Tohoku). Based on the remarkable descriptive analysis of these theories produced in the book cited above, I will try to question their more directly mathematical-and -philosophical significance, from the point of view of their possible unity.

- **Bibliography:** *Séminaire de Géométrie Algébrique du Bois-Marie SGA41/2* par P. Deligne avec la collaboration de Jean-François Boutot, Alexandre Grothendieck, Luc Illusie et Jean- Louis Verdier

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